Single cash flow Analysis

TIME VALUE OF MONEY

A. Simple Interest

$$I = Pin$$

I - interest

P - present value (principal)

i = simple interest rate

n = no. of period, in ref. to a year

$$F = P + I = P + Pin = P(1 + in)$$

F = accumulated or future value

Types:

a) Ordinary simple interest : 1 mo = 30 days

b) Exact simple interest : get the exact dates

Problem: Find the accumulated amount of ₱20,000 for __ at 5% simple interest

a) 9 months

b) 4 years & 4 months

Answer:

a) $F = 20000\left[1 + (0.05)\left(\frac{9}{12}\right)\right]$

$= ₱20,750$

b) $F = 20000\left[1 + (0.05)\left(\frac{52}{12}\right)\right]$

$= ₱24,333.\overline{33}$

Find the interest earned from a ₱10,000 deposit made from Jan 23, 2000 to Nov 5, 2000 at 5% simple interest.

Jan = 8
Feb = 29
Mar = 31
Apr = 30
May = 31
Jun = 30
Jul = 31
Aug = 31
Sept = 30
Oct = 31
Nov = 5

TOTAL:
288 days

$I = 10,000(0.05)\left(\frac{288}{366}\right) = ₱393.44$

COMPOUND INTEREST

$$F = P(1+i)^n \quad ; \quad P = F(1+i)^{-n}$$

n = period, ref. to a year

i = interest rate per year

$i = \frac{r}{m}$ → rate
m → no. of compounding periods in a year

Ex. Find the accumulated amount of ₱20,000 invested at ____ for a period of 8 years:

a) 4% compounded semiannually
b) 4% compounded quarterly
c) 4% compounded monthly
d) 4% compounded annually
e) 4% compounded weekly
f) 4% compounded daily
g) 4% compounded continuously

a) 4% cpd. s.a.

$$F = 20{,}000\left(1 + \frac{0.04}{2}\right)^{2\times 8} = 27{,}455.71$$

b) 4% cpd. quarterly

$$F = 20{,}000\left(1 + \frac{0.04}{4}\right)^{4\times 8} = 27{,}498.81$$

c) 4% cpd. monthly

$$F = 20{,}000\left(1 + \frac{0.04}{12}\right)^{12\times 8} = 27{,}527.90$$

d) 4% cpd. annually

$$F = 20{,}000\left(1 + \frac{0.04}{1}\right)^{1\times 8} = 27{,}371.38$$

e) 4% cpd. weekly

$$F = 20{,}000\left(1 + \frac{0.04}{52}\right)^{52\times 8} = 27{,}539.17$$

f) 4% cpd daily

$$F = 20{,}000\left(1+\frac{0.04}{365}\right)^{365\times 8} = 27{,}542.07$$

g) CONTINUOUS COMPOUNDING

$$F = Pe^{rn}$$

$$F = 20{,}000\,e^{(0.04)(8)} = 27{,}542.52$$

Find the time for a ₱50,000 deposit to double itself at 5% compounded quarterly.

$P = 50{,}000$

$F = 2(50{,}000) = ₱100{,}000$

$F = 2P$

$$2P = P\left(1+\frac{0.05}{4}\right)^{4n}$$

$n = 13.95$ years

NOMINAL & EFFECTIVE RATES

1) Nominal rate - rate is specified at compounding periods less than a year

2) Effective rate - rate is specified at annual compounding period.

Problem:

1) Convert a nominal rate of 6% compounded monthly to a nominal rate compounded semi-annually? What is its effective rate?

6% cpd. monthly = ____ cpd. s.a

F at 6% cpd mo = F at ____ cpd. s.a.

at 1 yr: $\cancel{P}\left(1+\frac{0.06}{12}\right)^{12\times1} = \cancel{P}\left(1+\frac{r}{2}\right)^{2\times1}$

$r = 6.07\%$

6% cpd monthly = ____ % effective

F at 6% cpd. mo = F at ____ eff

at 1 yr: $\cancel{P}\left(1+\frac{0.06}{12}\right)^{12\times1} = \cancel{P}\left(1+\frac{r}{1}\right)^{1\times1}$

$r = 6.17\%$ effective

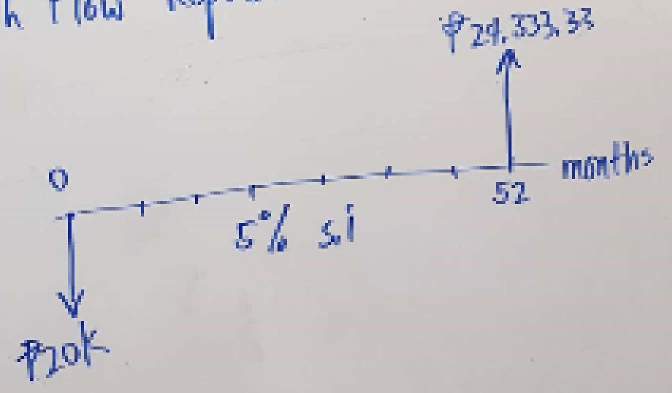
CASH FLOW DIAGRAM

↑ - cash inflow (+)

↓ - cash outflow (-)

Ex: ₱20,000 deposited at 5% simple interest will accumulate to ₱24,333.33 at the end of 4 yrs & 4 months.

Cash Flow Representation



Problem:

Carl, in preparation for his future milestones made the ff. investment deposits:

Present - ₱100k

After 6 months - ₱30k

After 1.5 yrs from present - ₱50k

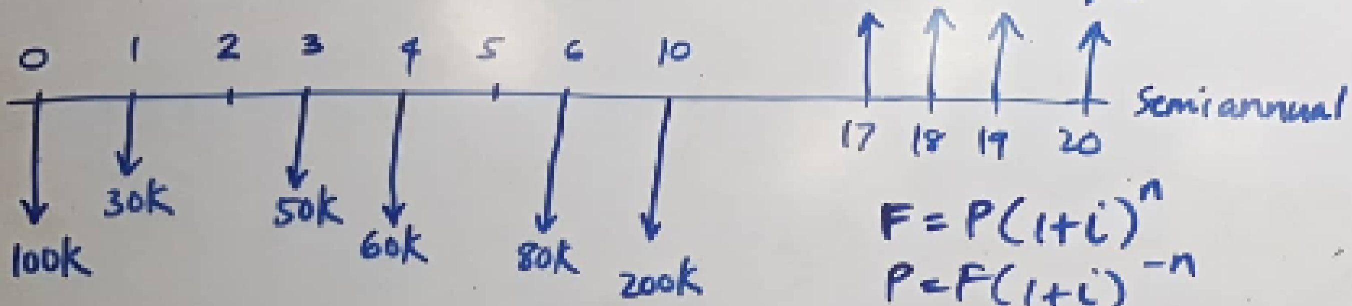
After 2 yrs. from present - ₱60k

After 3 yrs. from present - ₱80k

After 5 yrs from present - ₱200k

Ten yrs. from now, Carl plans to have his wedding ceremony. How much can he equally withdraw for 4 semiannuals if investment is at 5% cpd semiannually? The 4 equal withdrawals will be made at the end of the 17th semiannual period.

r = 5% cpd. s.a.



$F = P(1+i)^{n}$

$P = F(1+i)^{-n}$

Cash inflows = Cash outflows

Focal point: PRESENT (Ø)

$$100k\left(1+\frac{0.05}{2}\right)^{0-0} + 30k\left(1+\frac{0.05}{2}\right)^{0-1} + 50k\left(1+\frac{0.05}{2}\right)^{0-3}$$

$$+ 60k\left(1+\frac{0.05}{2}\right)^{0-4} + 80k\left(1+\frac{0.05}{2}\right)^{0-6} + 200k\left(1+\frac{0.05}{2}\right)^{0-10}$$

$$= x\left(1+\frac{0.05}{2}\right)^{0-17} + x\left(1+\frac{0.05}{2}\right)^{0-18} + x\left(1+\frac{0.05}{2}\right)^{0-19} + x\left(1+\frac{0.05}{2}\right)^{0-20}$$

X = ₱179,656.69

FOCAL PT: End of 5^{th} semiannual

$$100k\left(1+\frac{0.05}{2}\right)^{5-0}+30k\left(1+\frac{0.05}{2}\right)^{5-1}+50k\left(1+\frac{0.05}{2}\right)^{5-3}+60k\left(1+\frac{0.05}{2}\right)^{5-4}$$

$$+80k\left(1+\frac{0.05}{2}\right)^{5-6}+200k\left(1+\frac{0.05}{2}\right)^{5-10}=x\left(1+\frac{0.05}{2}\right)^{5-17}+x\left(1+\frac{0.05}{2}\right)^{5-18}$$

$$+x\left(1+\frac{0.05}{2}\right)^{5-19}+x\left(1+\frac{0.05}{2}\right)^{5-20}$$

$$X=₱179,656.69$$

FOCAL PT: End of 30^{th} semiannual

$$100k\left(1+\frac{0.05}{2}\right)^{30-0}+30k\left(1+\frac{0.05}{2}\right)^{30-1}+50k\left(1+\frac{0.05}{2}\right)^{30-3}+60k\left(1+\frac{0.05}{2}\right)^{30-4}$$

$$+80k\left(1+\frac{0.05}{2}\right)^{30-6}+200k\left(1+\frac{0.05}{2}\right)^{30-10}=x\left(1+\frac{0.05}{2}\right)^{30-17}+x\left(1+\frac{0.05}{2}\right)^{30-18}$$

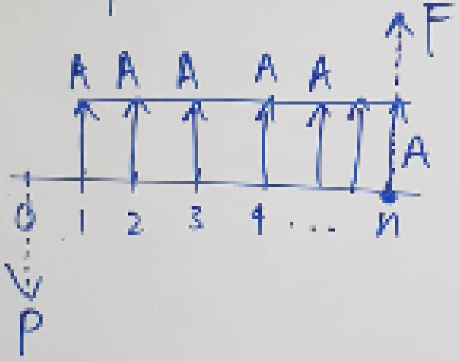
$$+x\left(1+\frac{0.05}{2}\right)^{30-19}+x\left(1+\frac{0.05}{2}\right)^{30-20}$$

$$X=₱179,656.69$$

Periodic equal cash flow Analysis (Annuities)

Annuity - serie of equal periodic payments

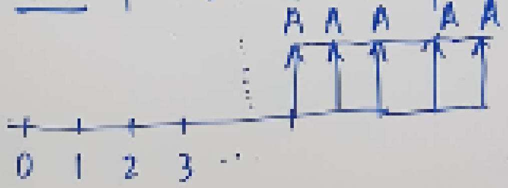
1) Ordinary annuity - first payment made at the <u>END</u> of the first period.



$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

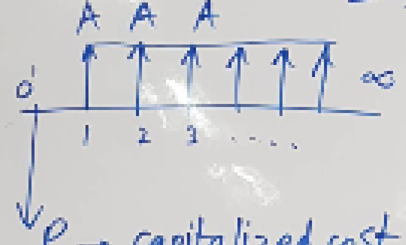
$$P = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

2) Deferred annuity - first payment made at the <u>END</u> of the deferred period.



NOT

3) Perpetuity - series of equal payments, ending indefinitely.

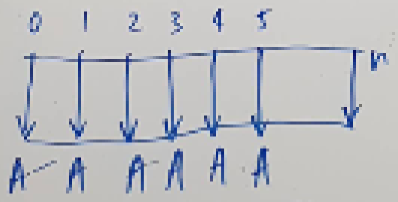


$$P = \frac{A}{i}$$

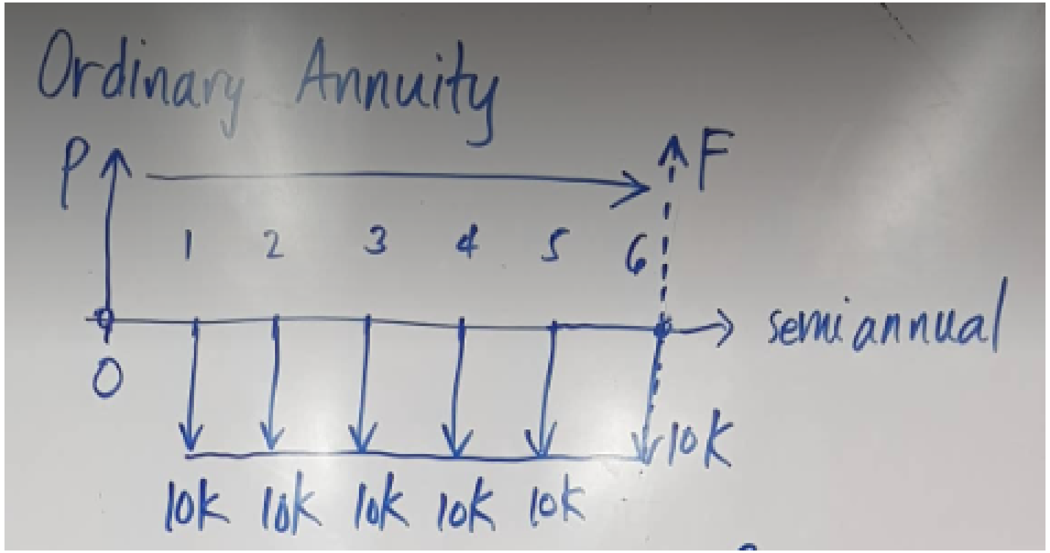
P → capitalized cost

We don't solve for F for perpetuity.

4) Annuity due - first payment made at the BEGINNING of the first period.



NOTE: Unless specified, all payments will be made at the END of the period.



- Find F and P if the rate is at 8% compounded semiannually using (a) single cash flow analysis (b)annuity formula
- Now, find F and P if the rate is at 8% compounded monthly.

Using single cash flow analysis:

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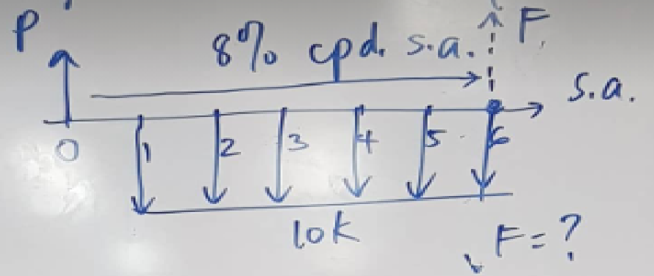


Solve for P: Focal pt: PRESENT (∅)

$$P\left(1+\frac{0.08}{2}\right)^{0-0} = 10k\left(1+\frac{0.08}{2}\right)^{0-1} + 10k\left(1+\frac{0.08}{2}\right)^{0-2}$$

$$+\ 10k\left(1+\frac{0.08}{2}\right)^{0-3} + 10k\left(1+\frac{0.08}{2}\right)^{0-4} + 10k\left(1+\frac{0.08}{2}\right)^{0-5}$$

$$+\ 10k\left(1+\frac{0.08}{2}\right)^{0-6} \qquad P = 52,421.37$$

Solve for F: $F = P(1+i)^n = 52,421.37\left(1+\frac{0.08}{2}\right)^{6-0}$

$$F = 66,329.75$$

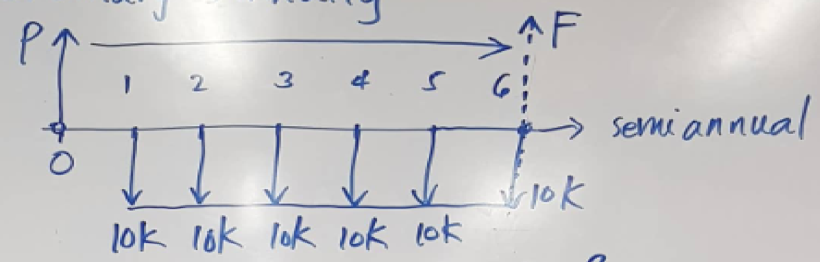
OR Solve for F: Focal pt: End of the 6th semiannual

$$F\left(1+\frac{0.08}{2}\right)^{0-0} = 10k\left(1+\frac{0.08}{2}\right)^{6-1} + 10k\left(1+\frac{0.08}{2}\right)^{6-2}$$

$$+\ 10k\left(1+\frac{0.08}{2}\right)^{6-3} + 10k\left(1+\frac{0.08}{2}\right)^{6-4} + 10k\left(1+\frac{0.08}{2}\right)^{6-5} + 10k\left(1+\frac{0.08}{2}\right)^{6-6}$$

$$F = 66,329.75$$

Ordinary Annuity



F = ?

Using annuity formula:

$$F = A\left[\frac{(1+i)^{n}-1}{i}\right]$$

$$= 10k\left[\frac{\left(1+\frac{0.08}{2}\right)^{6-0}-1}{0.08/2}\right]$$

$$F = 66{,}329.75$$

$$P = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 10k\left[\frac{1-\left(1+\frac{0.08}{2}\right)^{0-6}}{0.08/2}\right]$$

$$P = 52{,}421.37$$

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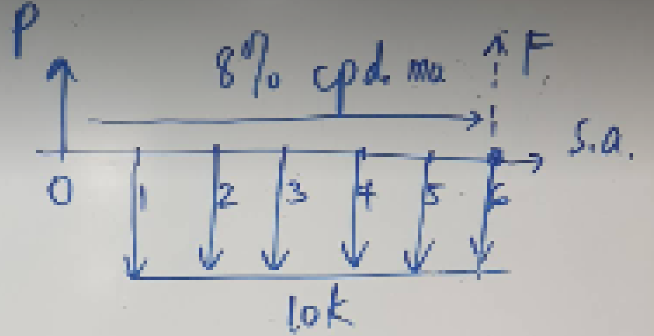


b) rate of 8% cpd. monthly

change the rate to semiannually:

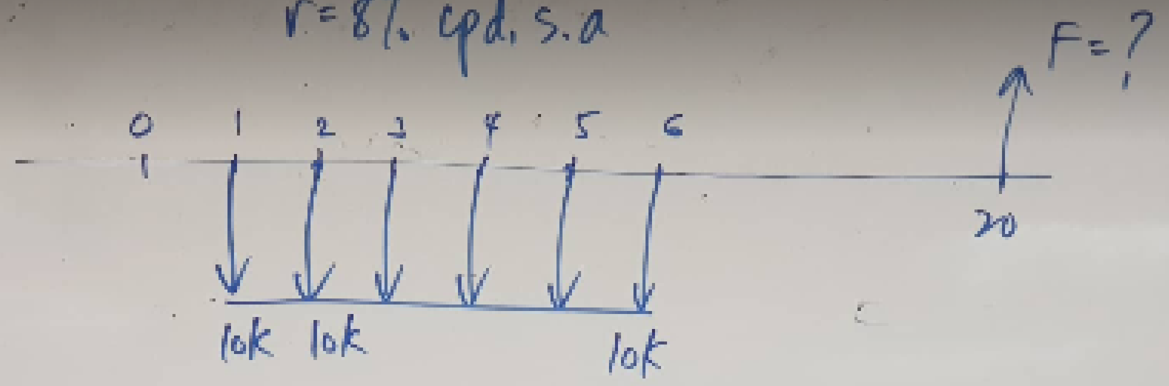
$$\left(1+\frac{0.08}{12}\right)^{12} = \left(1+\frac{r}{2}\right)^{2}$$

$$r = 8.13\% \text{ cpd. semiannually}$$

$$F = 10k\left[\frac{\left(1+\frac{0.0813}{2}\right)^{6-0}-1}{0.0813/2}\right] = 66{,}438.23$$

$$P = 10k\left[\frac{1-\left(1+\frac{0.0813}{2}\right)^{0-6}}{0.0813/2}\right] = 52{,}310.62$$

r = 8% cpd. s.a



Find F at the end of the 20th semiannual.

Single cash flow. Focal pt: End of SA 20

$$F\left(1+\frac{0.08}{2}\right)^{0-0} = 10k\left(1+\frac{0.08}{2}\right)^{20-1} + 10k\left(1+\frac{0.08}{2}\right)^{20-2}$$

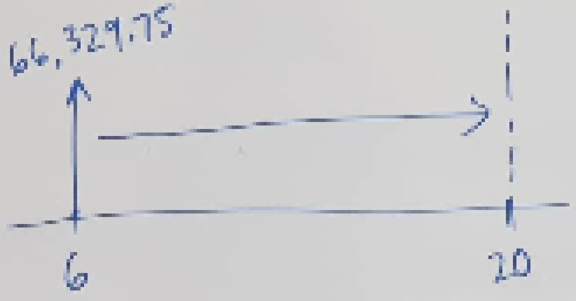
$$+ 10k\left(1+\frac{0.08}{2}\right)^{20-3} + 10k\left(1+\frac{0.08}{2}\right)^{20-4} + 10k\left(1+\frac{0.08}{2}\right)^{20-5} + 10k\left(1+\frac{0.08}{2}\right)^{20-6}$$

$$F = 114{,}861.67$$

Using annuity formula:

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = 10k\left[\frac{\left(1+\frac{0.08}{2}\right)^6 - 1}{0.08/2}\right]$$

$$F_{6th} = 66,329.75$$



F at 20: $F_{20} = 66,329.75\left(1+\frac{0.08}{2}\right)^{20-6}$

$= 114,861.67$